

Two mechanisms, three stages of the localization of light in a disordered dielectric structure with photonic band gaps

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We study the localization of light in the disordered dielectric structures derived from periodic systems with photonic band gaps. It is directly shown by field pattern that with *moderate disorder* there exist strongly localized states in the original gap. The localization-delocalization-localization trilogy discovered in this paper embodies the concept “interplay between the order and disorder” that has been previously suggested, and it is pointed out that the complete understanding of this complex localization behavior should take into account two localization mechanisms with different ranges of coherence. The difference between the two localization mechanisms is demonstrated by the different localization behaviors of the TM and TE modes. The *C* number is introduced to show the localization centers are frequency dependent. The frequency distribution of localization centers is found to vary with the degree of disorder. [S0163-1829(99)00638-4]

I. INTRODUCTION

Recent years have witnessed a consistent interest in photonic band structure.¹ Two themes are present in this field of research. The potential for applications to various devices, e.g., high efficiency laser² and antenna,³ has attracted most experimental researchers' attention, while impetus from the theoretical side lies in its candidacy for an ideal structure to test the theory of localization.⁴

Following the pioneering steps of Anderson⁵ and Mott,⁶ numerous researchers have been devoted to the study and observation of the electron localization in disordered solids. Some general features of localization are well established now.⁷ However, due to the existence of electron-electron interaction and electron-phonon interaction, theoretical predictions often meet with difficulties in accounting for the experimental data definitively. John first explored the possibility of observing localized states of light in a dielectric medium,⁸ in which the complication of the interactions associated with electrons is avoided so the further study of localization⁹ in such systems is facilitated. Experiments quickly verified the existence of weak localization in the form of coherent backscattering.¹⁰ However, for the implementation of strong localization of light, there is another serious problem: the effective energy of the wave equation of light in an inhomogeneous medium is always positive, and, even worse, the energy of the photons is always higher than the potential barriers.¹¹ After Yablonovitch first proposed a three-dimensional dielectric structure with photonic band gaps (PBG), John recognized⁴ that a moderate disordered perturbation of this structure may provide the key to the predictable and systematic observation of strong localization of light. The gist of his theory is that the underlying superlattice provides a band gap for the frequency range of localization because of the remnant geometric Bragg resonances. Though studies along this theoretical line^{12–14} have been car-

ried out by several groups, some fundamental questions have remained unsolved as discussed below. Many studies (experimental and theoretical) have been conducted on the disorder in a homogeneous background,^{15,16} instead of in a superlattice. Their results cannot be generalized to localization in PBG materials because of the absence of Bragg scattering in the homogeneous background. Therefore the problem of strong localization in a moderately disordered lattice is far from fully understood. For example, the only known embodiment of the “interplay between order and disorder”⁴ is that the localization is present at moderate, rather than at high, degrees of disorder, but will this interplay bring some other drastic changes to the whole picture of localization? Further, John has argued that the localization mechanism due to the remnant geometric Bragg resonances in a dielectric structure is a new one.⁴ Then the question remains whether the localization mechanism with which we are familiar in the Anderson model is also present in a dielectric structure? If it is, how will these two mechanisms interact with each other, and what will be interesting to note about the coexistence of these two mechanisms?

We are not well equipped to answer these questions. The analytic treatment has proved to be difficult. The well-established formalism¹⁷ assumes a low concentration of scatterers or a low degree of disorder. However, as we shall see, the most interesting localization behaviors in PBG materials occur at moderate disorder. On the numerical side, calculations with dimensions higher than one usually require prohibitive time and/or memory. So it is not surprising that these questions still reside in the category of the unsolved. Even John's original proposal (strongly localized states present at moderate disorder) has not been directly verified by numerical calculation.

However, Meade *et al.*¹⁸ have introduced a memory-economical computing scheme in their calculations of photonic bands and impurity energy levels. This method has

greatly enlarged the size of systems that we are able to calculate. But, applied to the supercell problem, this algorithm usually needs to calculate all the states with frequencies lower than the desired one, a requirement of orthogonalization. As the supercell grows larger and larger, the workload becomes intolerable. In this paper, a scheme for the supercell problem is developed to calculate a few eigenstates with frequencies closest to a specific value without the need to calculate those of the lower states.

With this method, we directly show by field pattern that, at moderate disorder, there exist strongly localized states inside the original photonic band gaps. We find that as the degree of disorder increases, the states in or near the PBG first localize, then delocalize, then finally localize again. This localization trilogy is clearly shown due to the cooperation of two localization mechanisms, which are distinguished by their different ranges of coherence. Due to their different sensitivities to these two localization mechanism, the TM and TE modes exhibit different localization behaviors. Our analysis answers, simultaneously, the problems proposed at the end of the second paragraph.

The partition number¹⁹ commonly used proves to be a very useful measure to analyze wave functions in localization problems. However, sometimes it does not contain all the desired information. To study the nature of localization centers at different frequencies, a new measure, the C number, is introduced in our study. A sum rule involving the C number and partition number is obtained in this work. With the C number, we find that the sites a photon prefers to occupy are obviously related to frequency. The frequency distribution of localization centers established at moderate disorder is maintained till strong disorder, except in the frequency ranges around the original gaps.

The structure of this paper is as follows: In Sec. II, the problem and the computational scheme are described. In Sec. III, we introduce the C number and propose a sum rule involving the C number and partition number. In Sec. IV, first, we show the localization process plays a trilogy as the degree of disorder varies and discuss the underlying mechanism, then the distribution of C numbers with frequency is studied. Section V contains the conclusion.

II. COMPUTATIONAL SCHEME

In the photonic crystal, the dielectric constant $\epsilon(\mathbf{r})$ is a function of position. The electromagnetic field is determined by the solution of the Maxwell's equations. For the reason of continuity of the field vectors, it is convenient to solve these equations in terms of the magnetic field vector $\mathbf{H}(\mathbf{r})$. The wave equation is then

$$\nabla \times \left[\frac{1}{\epsilon(\mathbf{r})} \nabla \times \mathbf{H}(\mathbf{r}) \right] = \left(\frac{\omega}{c} \right)^2 \mathbf{H}(\mathbf{r}), \quad (1)$$

with the transverse field condition $\nabla \cdot \mathbf{H}(\mathbf{r}) = 0$. A typical two-dimensional dielectric superlattice is either a periodic array of dielectric columns in the air or a periodic array of air columns in the dielectric bulk. The centers of the columns can be arranged into any type of two-dimensional lattice. The lattice studied in this paper is a triangular lattice of dielectric columns in the air. There are two parameters in the definition of this periodic dielectric structure, the dielectric

constant of the dielectric material and the column radius. In this study, the dielectric constant is chosen to be $\epsilon_a = 13$, which is close to the value of gallium arsenide. The column radius is $0.32a$, where a is the lattice constant. For disordered systems, the supercell comprises 256 (16×16) unit cells and we calculate up to 16400 plane waves for each polarization. The relative deviation of column dielectric constants ϵ is uniformly distributed in the range $-W/2 < \epsilon/\epsilon_a - 1 < W/2$, where W indicates the degree of disorder.

Meade *et al.* first introduced an optimization algorithm for the calculation of photonic band structure.¹⁸ The optimization algorithm greatly reduces the required memory because it changes the wave functions at each step instead of changing the operators. Hence the operator need not be stored as it can be constructed impromptu at a reasonable expense of time by using the fast Fourier transform (FFT). Still, when applied to the supercell problem, the workload of any optimization algorithm is overwhelming because the states we are interested in are usually inside or near the gap. The orthogonalization process of the algorithm requires us to calculate all the states lower than the target state; therefore, one must calculate an entire band of states.

We have developed a method that enables us to calculate several states closest to a specific energy without calculating lower-energy states. Suppose the energy functional to be $\Psi^\dagger Q \Psi$, Q is the main operator, Ψ is the wave function. If the optimization object is $\Psi^\dagger (Q - E)^2 \Psi$ (E is the specified energy) rather than the energy functional, the eigenstates obtained first are those with energies nearest to E . We note the operator $(Q - E)^2$ can also be constructed impromptu with the FFT algorithm; therefore, this method leaves intact all the time- and memory-saving features of the previous algorithm. With this method, we have performed a systematic calculation of localization of electromagnetic waves in two-dimensional structures. For every frequency studied, five states nearest to this frequency are calculated for TM (with the magnetic field vector lying in plane) and TE (with the magnetic field vector normal to the plane) modes, respectively. Then for each state, we define the coarse-grained field $\tilde{H}(i, j)$ by integrating the square modula of the magnetic field in each unit cell Ω_{ij} ,

$$\tilde{H}^2(i, j) = \int_{\Omega_{ij}} |\mathbf{H}(\mathbf{r})|^2 d\mathbf{r}. \quad (2)$$

These coarse-grained spatial field patterns are recorded on the hard disk to save space.

III. A NEW MEASURE: C NUMBER

The partition number introduced by Dean¹⁹ is a powerful tool to analyze the large volume of data obtained from the *ab initio* calculation. The partition number is defined by

$$P = \left[\sum_i |\Psi_i|^4 \right]^{-1}, \quad (3)$$

where Ψ_i is the normalized wave function. It roughly gives the number of sites where the mode is significantly excited. Thus $1/P$ indicates the degree of localization of a state. However, often one needs to obtain more information about the character of the microscopic states than given solely by the

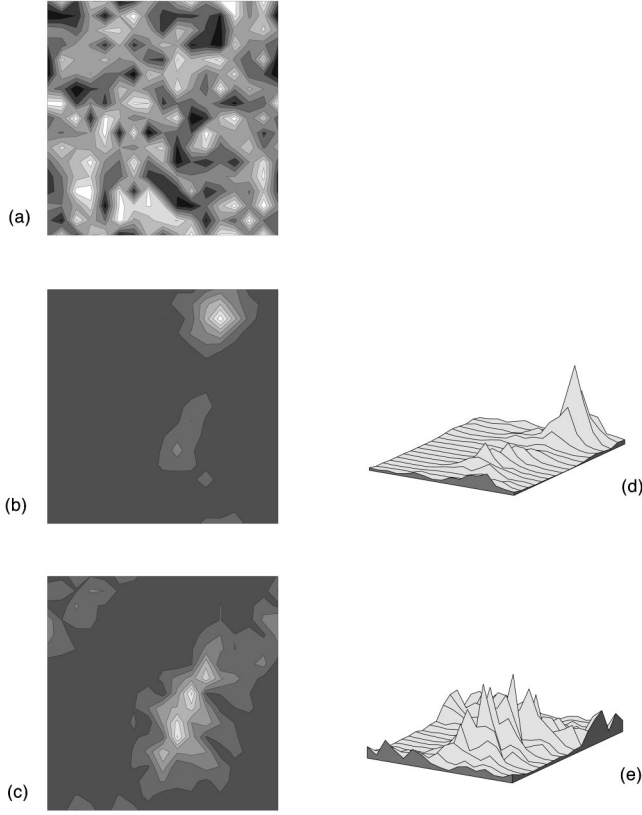


FIG. 1. The configuration of the system and the field patterns of some localized states. For convenience, the triangular lattice is plotted as a square one. In the contour maps, the white color indicates the highest value, whereas the black color indicates the lowest. (a) The configuration of the system is presented by the contour map of the cell-average dielectric constants. (b) and (d) A strongly localized TM state, two different ways of plotting; (c) and (e) strongly localized TE states, two different ways of plotting. (a), (b), and (c) clearly exhibit the relation ship between the localization centers of the two states and the configuration of the system. For TM state, $\omega a/2\pi c = 0.228$, $P = 11.67$, $C = -0.170$, at $W = 1.0$. For the TE state, $\omega a/2\pi c = 0.331$, $P = 32.9$, $C = -0.224$, at $W = 0.4$.

partition number. For the localization problem, one character we are interested in is the nature of the localization center(s) of a localized state. From the partition number one may at best get a feeling of how fast the wave function decays from these centers. But to find which kind of sites are preferred centers for a specific state, one needs another measure.

A close inspection of the spatial dielectric distribution and the field patterns of several localized states reveals the meaning of the new measure to be introduced.

Figure 1 (a) presents the distribution of the cell-average dielectric constants in 16×16 unit cells (for visual ease, the triangular lattice is drawn as a square one). We use the contour map because it best visualizes the relation between the average dielectric constant distribution and the localization centers. However, one should keep in mind that the actual average dielectric constant distribution is defined on a 16×16 discrete grid. In Figs. 1(b) and 1(d), a strongly localized TM state is drawn in two ways against the map of the dielectric constant distribution. This state is found for the strength of disorder $W = 1.0$, at the frequency $\omega a/2\pi c = 0.228$ (subsequently, all frequencies are given in the unit of $2\pi c/a$ in

this paper), with a partition number 11.7. Note that for the periodic system with our division of unit cells, the first gap of the TM mode ranges from 0.209 to 0.273, and the first TE gap is between 0.325 and 0.360. The localization center of this state corresponds to the low dielectric island near the upper edge, as one sees by comparing Figs. 1(a) and 1(b). The TE state shown in Figs. 1(c) and 1(e) is found for $W = 0.4$, at the frequency 0.331, with a partition number 32.9. Clearly, it is localized around a chain of low dielectric constant islands in the middle of the system. However, for less localized states, the field patterns are not as simple as in Figs. 1(b) and 1(c), and the correspondence between the field patterns and the dielectric distribution is not easy to detect by direct inspection. In most cases, one needs to introduce the C number defined below to measure this correspondence.

Suppose we have a coarse-grained magnetic field $\tilde{H}(i,j)$ defined on the lattice level (so i,j run all over lattice sites). The C number is defined by

$$C = \rho(\epsilon, \tilde{H}^2), \quad (4)$$

where $\epsilon(i,j)$ is the average dielectric constant in the unit cell Ω_{ij} and the correlation coefficient $\rho(X,Y)$ between two random variables $X(i,j), Y(i,j)$ defined on lattice sites is given by the well-known equations

$$\rho(X,Y) = \frac{\text{cov}(X,Y)}{\sigma_X \sigma_Y},$$

$$\text{cov}(X,Y) = \frac{1}{N-1} \sum_{i,j} [X(i,j) - \bar{X}][Y(i,j) - \bar{Y}], \quad (5)$$

$$\sigma_X = \sqrt{\text{cov}(X,X)},$$

where N is the total number of sites, and $\bar{X}$ and $\bar{Y}$ are the mean values of X and Y on the lattice. For the TM state given in Figs. 1(b) and 1(d), the C number is -0.170 . This negative C number indicates the coarse-grained field has a large amplitude around some cells containing columns with low dielectric constants. The C number for the TE mode shown in Figs. 1(d) and 1(e) is -0.224 . According to the location of their frequencies in the gap, these two states may be called acceptorlike states.²⁰ These kinds of states are associated with negative C numbers.

A sum rule can be obtained for the C number and the partition number. When the disorder is not very strong, a large part of the original gap survives. Then the single band approximation holds (as assumed in the Anderson model), which means that a mode in a certain band of the disordered system is a linear combination of states in the corresponding band of the periodic system. This implies

$$\sum_n \psi_n^+(\mathbf{r}) \psi_n(\mathbf{r}') = \sum_k \phi_k^+(\mathbf{r}) \phi_k(\mathbf{r}'), \quad (6)$$

where $\phi_k(\mathbf{r})$ and $\psi_n(\mathbf{r})$ are the states of periodic and disordered systems, respectively; the summations of k and n extend over a single band in the corresponding systems. Let $\mathbf{r} = \mathbf{r}'$, and integrate the right-hand side in any unit cell, and one finds that the integral is a constant, m (m is a positive

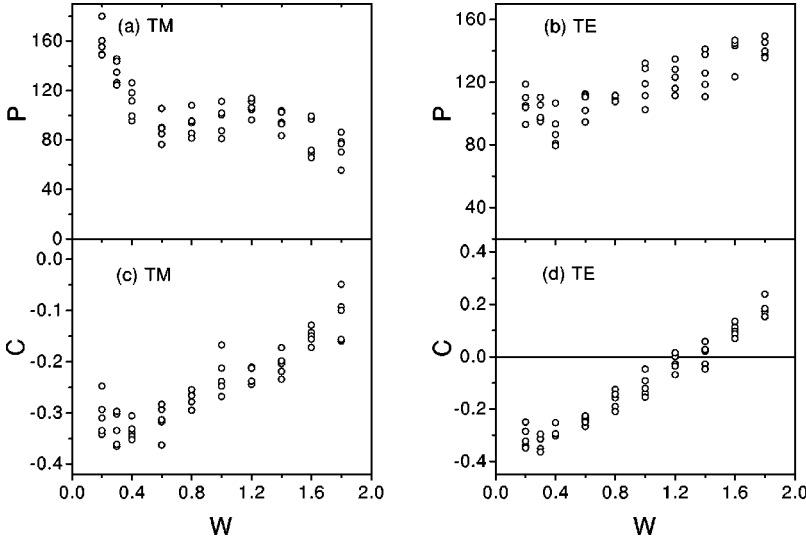


FIG. 2. The variation of the partition numbers and the C numbers of the states nearest to the frequency slightly above the lower edge of the gap with increasing degree of disorder, for TM ($\omega a/2\pi c=0.212$) and TE ($\omega a/2\pi c=0.329$) modes, respectively.

integer), independent of the position of the cell. For our coarse-grained field in the disordered system this means

$$\sum_n \tilde{H}_n^2(i,j) = m. \quad (7)$$

Another useful equation is

$$\sigma_{\tilde{H}^2}^2 = \frac{1}{N-1} \left(\frac{1}{P} - \frac{1}{N} \right). \quad (8)$$

Now one readily proves

$$\sum_n C_n \sqrt{\frac{1}{P_n} - \frac{1}{N}} = \frac{\sqrt{N-1}}{\sigma_\epsilon} \sum_{i,j} [\epsilon(i,j) - \bar{\epsilon}] \sum_n \tilde{H}_n^2(i,j) = 0, \quad (9)$$

where C_n and P_n are the C number and partition number corresponding to $\tilde{H}_n(i,j)$. In terms of the energy spectrum, we have

$$\int \rho(E) C(E) \sqrt{\frac{1}{P(E)} - \frac{1}{N}} dE = 0. \quad (10)$$

Several points should be noted for this sum (or integral). First, fully extended states do not contribute to this sum. Second, for the strongly localized states, the C numbers may be quite small (compared with fairly localized states); however, their small partition numbers compensate for it so that their contribution to the sum is still considerable. Due to the enormous time required for calculation of an entire band of states, we only perform a rough check of the sum rule on a 8×8 lattice. For $W=0.2$, the sums in Eq. (9) divided by the corresponding sums of absolute values are 0.07 and 0.18 for the TM mode and TE mode, respectively. When we increase the iteration times by one third, the above values reduce to 0.05 and 0.14. We also note a convergence of the above values from a 4×4 lattice to an 8×8 lattice. This sum rule will be better satisfied on a 16×16 lattice, where the ratio of a typical interband component to a typical intraband component in a perturbed state is about 1% and 3% according to the energy difference denominators in the perturbation theory.

IV. RESULTS AND DISCUSSION

A. Localization trilogy

At first, we study the localization behavior of the states near some specific frequencies as the degree of disorder changes. As shown in Fig. 1, very strongly localized states are found in the original gap at *moderate* disorder ($W=1.0$ for the TM mode, and $W=0.4$ for the TE mode), as predicted by John.⁴

The localization process of the states shows a complex feature. It is well known that in the Anderson model, the states become more localized with an increasing degree of disorder.²¹ Recently, Freilikher *et al.*¹³ have found that for frequencies in the gap, there may exist an initial stage of enhancing transmission with an increasing degree of disorder before the final stage of attenuating transmission at strong disorder. This has also been noted in a two-dimensional system,¹⁴ though, for the range of disorder studied, the final attenuation range was not reached. The characteristic of this kind of enhancing transmission is that it occurs only in the gap and starts from infinitely weak disorder; hence it was attributed to the increasing of density of states (DOS) in the gap due to the presence of disorder. However, the question remains, how the localization of microscopic states evolves with increasing disorder. Specifically, does the variation of localization lengths of the states have the features corresponding to the results of the studies of transmission? We find the localization of states shows more complex process than that reflected by studies of transmission, and this process is explained by the coexistence of two localization mechanisms associated with different ranges of coherence.

Figure 2 presents the evolution of five states with frequencies slightly above the lower edge of the lowest gap for the TM and TE modes, respectively. It shows that the localization behavior, or more appropriately for this case, the localization-delocalization behavior of the TM mode can be described as a trilogy. When the degree of disorder W is less than 0.6, the localization of the states becomes more pronounced with increasing W . After that, the states begin to delocalize until $W=1.2$; then they show the localization trend again with a further increase in W . What we find is the delocalization of states begins at some intermediate disorder,

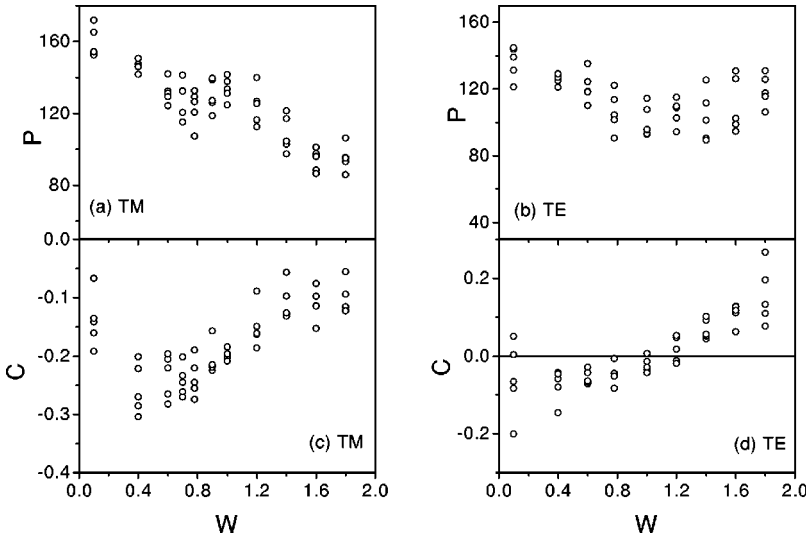


FIG. 3. The variation of the partition numbers and the C numbers of the states nearest to the frequency slightly below the gap with increasing degree of disorder for the TM mode ($\omega a/2\pi c = 0.202$) and the TE mode ($\omega a/2\pi c = 0.319$).

and the delocalization is also found in the original band (as shown later), where DOS is generally decreasing. However, this surprising trilogy can be well explained by considering the joint effect of two localization mechanisms. When disorder is weak, the remnant Bragg resonant scattering of the underlying superlattice results in localization of photons. As the dielectric constants of the columns deviate farther and farther from the original value, the states presented in Fig. 2 localize around some low dielectric islands as those shown in Fig. 1. The difference between the dielectric constants of these islands and the average periodic background increases as W increases, so the localized states feel stronger Bragg scattering from the underlying superlattice, with the effect of further confining them around those islands. But when the disorder is very strong, the remnants of the underlying superlattice are completely destroyed. Then the localization process enters into another range. Whereas the localization mechanism provided by Bragg scattering is a long-range coherent effect, the localization in the strongly disordered system is of the Anderson type, i.e., it is a local effect. The localization length is mainly determined by the local mismatch (the difference of dielectric constants between adjacent cells within several lattice constants), rather than by the deviation from the average background. But there must be a range of the degree of disorder where the Bragg-scattering effect is diminishing while the Anderson localization is enhancing. The Anderson localization is not as strong as that produced by Bragg scattering, at least in this intermediate range of disorder; so the states begin to delocalize. John argued⁴ that for weak disorder the wavelength entering the Ioffe-Regel²² condition should be the inverse of the fluctuation in the wave vector from the Bragg plane; then we propose that for very strong disorder this wavelength should be the free-photon wavelength. For intermediate degrees of disorder, neither of these two lengths meets the Ioffe-Regel condition (in fact, the first length is not well defined for fairly strong disorder), so the states delocalize.

For the TE mode, Fig. 2(b) shows that the localization-delocalization process ends in the second stage. The third stage will appear if one sets the average dielectric constant of the columns much larger, which provides greater local mismatch. There is a simple argument to explain why TE modes are resistant to the Anderson localization. The amplitudes of

the band-edge TE modes are more concentrated in the air region and this is more obvious in a system formed by dielectric columns in air.²³ In the structure treated here, the air region is connected through the entire system. From the viewpoint of percolation, this is unfavorable to localization, particularly to that of the Anderson type, because the percolation is a short free-path process, while the localization due to Bragg scattering depends on large-scale resonance. The different localization behavior of TM and TE modes thus reveals a major difference of the Bragg-type localization mechanism from the Anderson type.

The C numbers for this system are presented in Fig. 2(c) and 2(d). Generally, in the initial localization stage ($W \leq 0.6$ for the TM mode, and $W \leq 0.4$ for the TE mode) the magnitudes of C numbers increase, then remain nearly constant as the disorder increases. This is another proof to the effect of the Bragg scattering in this stage of localization: the states are becoming more localized around some low dielectric cells. However, there are many such cells in the system. As one state becomes more localized around only one, or several of them, the C number of the state will suffer a decrease. But because the amplitude of the mode at the localization centers is greatly enhanced, this decrease is not significant. Furthermore, this slight decrease of the magnitudes of C numbers at the end of the initial localization stage should not be confused with the obvious decrease in the delocalization stage, which is due to the diffusion of the mode amplitude into high dielectric constant cells. For the TE mode, the C numbers finally become positive as a result of the “invasion” of the upper band. The TE gap is much smaller than the TM gap, so it disappears (about $W=0.9$) very quickly as W increases. Then there is a mixing of the donorlike (associated with positive C numbers) and acceptorlike states; the C numbers tend to be neutral. In the end, the states of the upper band further move down, and crush onto the original lower band. These states (acceptorlike) maintain their positive C numbers till very large W , as shown in the next subsection.

We also studied the states at another frequency slightly below the gap. The delocalization stage for TM mode is not as pronounced in Fig. 3 (a) as it is in Fig. 2(a). This shows that the competition between the diminishing Bragg localiza-

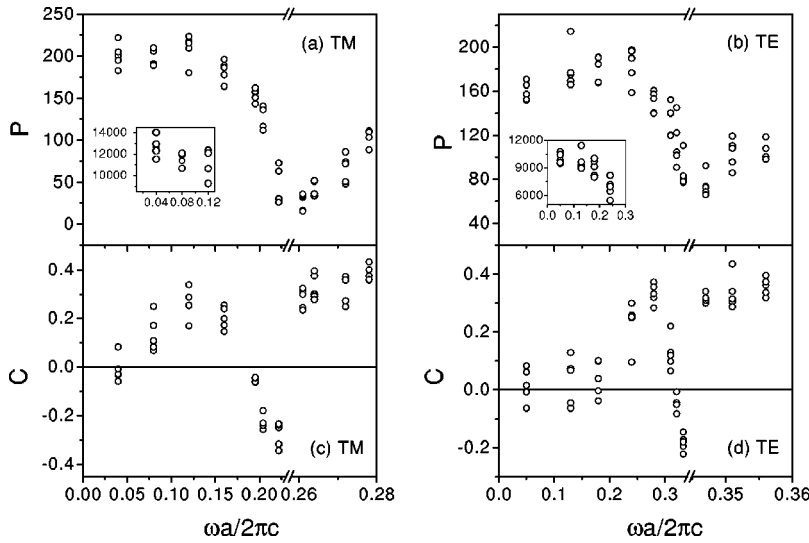


FIG. 4. The variation of the partition numbers and the C numbers with frequency, for the TM and TE modes, respectively ($W=0.8$). Note the axis break at the remnant gap and the different scales before and after the break. The insets in (a) and (b) show the partition numbers on the grid basis, for the lowest several frequencies. They show P increases as the frequency decreases in the long-wave range.

tion and the enhancing Anderson localization nearly reaches a balance in this frequency range. The delocalization is more obvious for the TE mode, which is due to the absence of the final localization stage, as in the gap. The graphs of C numbers for this frequency are nearly the same as those for the frequency inside the gap. For the TE mode, some C numbers are positive in this frequency range for very weak disorder. This is due to the distribution of C numbers with frequency as discussed in the next subsection.

B. Frequency distribution of C numbers

In this subsection, we fix W and study the localization feature at different frequencies. In Fig. 4, W is set to 0.8; at this intermediate degree of disorder the remnant gap for the TM mode is from 0.222 to 0.261, while for TE mode, it is from 0.341 to 0.347. One sees that there is a considerable drop of partition number at the band edge.

It is well known that the low-frequency modes should be well extended, but there seems some slight drop of P in this range. To solve this problem, we calculated the partition number on the entire grid, so that the maximum P is about 16 400. These kinds of partition numbers are plotted in the insets of Figs. 4(a) and 4(b). Clearly, these partition numbers increase consistently as the frequency decreases. The difference between the partition number on the lattice basis and that on the grid basis is due to the inner structure of the field pattern. In fact, for most states in the lowest band, the partition number on the lattice basis is mainly determined by the localization of the wave on the lattice level, and the trend of two kinds of partition numbers is found to agree well. However, when the frequency approaches zero, the coarse-

grained field is nearly uniform on the lattice level. Now the delocalization, or, more properly, the redistribution of the wave amplitude, occurs inside unit cells, particularly across the interfaces of the dielectric columns and the air region. But, generally, the localization problem is concerned with behavior on the lattice level (as in the Anderson model).

The C number distribution shows clearly the division between the donorlike and acceptorlike states by the gap. The donorlike states have positive C numbers and the acceptorlike states have negative C numbers. The discontinuity of the frequency distribution of the C numbers occurs at the gap. In the band, the C numbers vary continuously with frequency. This means the dielectric constants of the columns around which a state prefers to stay vary continuously with frequency. Deep in the lower band, the C number is positive, as one expects from the sum rule. Physically, this is due to the balance of the local DOS. The sum of the local DOS within each band should be constant in space, as indicated by Eq. (7), even in the disordered system. As the band-edge states preempt the DOS in the cells with low dielectric constant columns, there must be some states in this band concentrating in the cells with high dielectric constant columns. The C numbers for the long-wavelength region are distributed around zero, which is in accordance with the fact that the field is distributed uniformly in the system. When the degree of disorder is small ($W=0.1$), this region of positive C in Figs. 5(a) and 5(b) is not as obvious as in Figs. 4(c) and 4(d), respectively. This implies that for weak disorder the band-edge states do not occupy as much of the local DOS at the cells of low dielectric constant columns. This observation corroborates that the states at band edge for slightly disor-

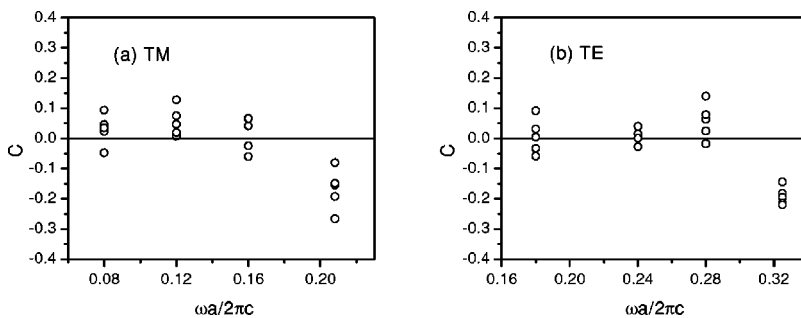


FIG. 5. The variation of the C numbers with frequency, for TM and TE modes, respectively ($W=0.1$). Both distributions are quite flat.

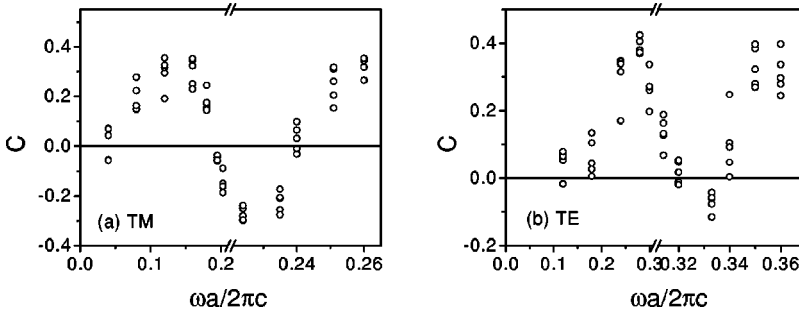


FIG. 6. The variation of the C numbers with frequency, for the TM and TE modes, respectively ($W=1.2$). Note the axis break and the different scales before and after the break. Note for this W , both the TM and TE gaps are closed. The axis breaks thus do not indicate remnant gaps.

dered system are not as strongly localized as those for intermediate disordered system. Figure 6 shows there is no substantial increase of C numbers in the positive region of the original lower band for the degree of disorder as high as $W=1.2$. As we have seen in Fig. 2, when W is about 0.6 for the TM mode and 0.4 for the TE mode, the localization due to the Bragg scattering begins to fade away, whereas this kind of localization was growing stronger consistently for lower values of W . One distinct feature of this Bragg-type of localization is the strong localization of states at the band edge and, as revealed here, the accompanying positive C number region at certain lower frequency ranges, which implies the localization of the states around the cells with high dielectric constants in this lower frequency range.

The picture is now clear. The localization center distribution is established in the weak disorder range by long-range coherence. However, in the strong disorder range, this distribution is modified, or destroyed particularly in some frequency ranges (near original gaps). When the disorder is very strong, the localization centers separated in space are no longer connected by long-range coherence; therefore, these centers do not necessarily have similar dielectric constants. In fact, for strong disorder, depending on the surrounding cells, a cell with a column of a high dielectric constant may have the same local energy level as the one with a low dielectric constant. As the disorder increases, the possibility increases of two disparate centers with the same local energy level arising within the localization length of each other. Then some states having disparate localization centers may

be present. This is the microscopic interpretation of the presence of neutral C numbers in the original gap.

V. CONCLUSION

We numerically studied the localization of light in the disordered dielectric structures derived from periodic systems with PBG. It was directly shown by field pattern that there existed strongly localized states in the original gap. The three-stage localization-delocalization process verifies the “interplay between the order and disorder” that has been suggested for a long time. We further point out that the trilogy arises because of different ranges of coherence for two kinds of localization mechanisms. Due to their different sensitivities to the two localization mechanisms, the TM and TE modes exhibit different localization behaviors. A measure, the C number, was introduced to gain a deeper insight into the nature of localization. It was found with this number that the localization centers are not random, but that states prefer to localize at a specific kind of cell in a specific frequency range.

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