

Longitudinal and Angular Dispersions in Photonic Crystals: A Synergistic Perspective on Slow Light and Superprism Effects*

Ryan A. Integlia, Weiwei Song, Jun Tan, and Wei Jiang*

Department of Electrical and Computer Engineering, and Institute for Advanced Materials, Devices, and Nanotechnology, Rutgers University, 94 Brett Road, Piscataway, NJ 08854, USA

The slow-light effect and the superprism effect are two important effects in photonic crystal structures. In this paper, we will review some of our recent works on the fundamental physics and device applications of these two effects. We will present a synergistic perspective that examines these two effects as a whole. Apparently, the slow light effect is due to the dispersion of a photonic crystal along the direction of light propagation, namely the longitudinal direction, and the superprism effect is related to angular dispersion. However, a deep analysis will show that the superprism effect has an elusive dependence on the longitudinal dispersion as well. Some subtle connections and distinctions between the slow-light effect and the superprism effect will be revealed through our physical analysis. This allows us to treat these two effects under a common theoretical framework. As an example, we will apply this framework to make a direct comparison of the slow-light optical phase array approach and the superprism approach to beam steering applications. Dispersive effects are frequently accompanied by high optical loss and/or narrow bandwidths. We will discuss these issues for both longitudinal and angular dispersions. For the slow light effect, we will give a simple proof of the scaling of fabrication-imperfection related random scattering losses in a slow-light photonic crystal waveguide. Similar to the bandwidth-delay product for the longitudinal dispersion, we will introduce a simple, yet fundamental, limit that governs the bandwidth and angular sensitivities of the superprism effect. We will also discuss the application of the slow-light effect to making compact silicon optical modulators and switches.

Keywords: Photonic Crystals, Slow Light, Superprism, Dispersion, Optical Loss, Bandwidth, Silicon Photonics.

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1. INTRODUCTION

Photonic crystals possess a wide range of extraordinary properties that are absent in conventional materials. First and foremost, the periodic structure of a photonic crystal causes photonic bands and bandgaps to form on the frequency spectrum of photons. Therefore, photonic crystals with photonic bandgaps can serve as "perfect mirrors" to confine light in small dimensions, forming ultracompact waveguides and cavities. On the other hand, there are other technologies that can also provide tight confinement of light. Compared to these alternative technologies, the uniqueness of photonic crystal-based waveguides and cavities often comes from the fact that the periodic structure of a photonic crystal provides some *additional* distinctive opportunities to modify the spectral property of light, leading to many dispersive effects with a wide range of applications. For example, in a photonic crystal waveguide, the

*Author to whom correspondence should be addressed.

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dispersion relation, $\omega(k)$, generally has a portion where its slope tends to zero, implying a vanishing group velocity. Such a slow light effect, together with the tight light confinement provided by the photonic bandgap, leads to extraordinary enhancement of phase shift and time delay in such a waveguide.¹ We shall emphasize that some of these dispersion effects can be significant in their own right, without the presence of tight light confinement. For

example, the superprism effect² causes the beam propagation angle inside a photonic crystal to be highly sensitive to the wavelength of light. Essentially, this effect amounts to significantly enhanced angular dispersion. In the superprism effect, the light propagation is not confined. Moreover, we will show that the superprism effect does not necessarily appear near a bandgap (or at a bandedge). It can indeed appear in the midst of a photonic band, thanks



Ryan Integlia received his M.S. in 2008 jointly from Rutgers Civil and Environmental Engineering Department and Electrical and Computer Engineering Department studying the subject of wireless sensor networks. He is currently researching the subjects of the slow light effect and the superprism effect. He received his bachelor's degree in 2001 from Rutgers University's Electrical and computer Engineering Department, graduating with high honors. Ryan Integlia was also a recipient of the NSF's IGERT Traineeship, the Center for Advanced Infrastructure and Technology Fellowship, among other honors.

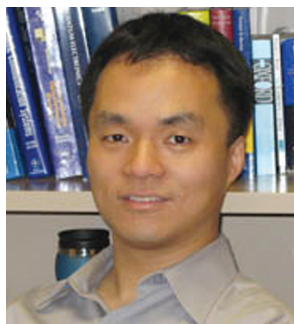
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Weiwei Song received his M.S. in 2008 from the Department of Physics of Nanjing University, China. He is currently pursuing his Ph.D. degree in the Department of Electrical and Computer Engineering of Rutgers University and working in the group of Professor Wei Jiang. His research involves silicon photonic crystal structures and devices, with a focus on the simulation and characterization. In 2005, Weiwei graduated from Nanjing University with a B.S. in Condensed Matter Physics.



Jun Tan received his B.S. in 2005 from Nanjing University, P. R. China, and his M.S. in 2008 from Shanghai Institute of Technical Physics, Chinese Academy of Sciences. Currently, he is pursuing his Ph.D. in the Department of Electrical and Computer Engineering, Rutgers University. In Dr. Jiang's group, his research is focused on device fabrication and characterization.



Wei Jiang received the B.S. degree in physics from Nanjing University, Nanjing, China, in 1996, and the M.A. degree in physics and the Ph.D. degree in electrical and computer engineering from the University of Texas, Austin, in 2000 and 2005, respectively. He held research positions with Omega Optics, Inc., Austin, Texas from 2004 to 2007. Since September 2007, he has been an assistant professor in the department of electrical and computer engineering of Rutgers University, Piscataway, NJ. His doctoral research made a contribution to the fundamental understanding of the wave coupling, transmission, and refraction at a surface of a periodic lattice. At Omega Optics, he led a research project to the successful demonstration of the first high-speed photonic crystal modulator. He also recognized a scaling law for the current density of high speed silicon electro-optic devices. His current research interests include photonic crystals, silicon photonics, optical interconnects, and beam steering. He received Ben Streetman Prize of the University of Texas at Austin in 2005.

to the high symmetry of a photonic crystal structure. The presence of a photonic bandgap is not a necessary condition for the superprism effect.

In this paper, we will review some of our recent theoretical and experimental works concerning these dispersive effects. In these works, our emphasis was placed on finding the general, *quantitative* physical laws governing these effects. For example, in the first superprism experiment, it was found that the photonic crystal could enhance the angular dispersion or sensitivity by 500 times;² later numerical simulations and experimental works reported varying enhancement factors.^{3–7} However, a rigorous, compact mathematical form of the physical law that can express this enhancement factor in terms of the photonic band parameters is missing. As such, while we can easily obtain an instance of high sensitivity structures, we can not systematically predict the trend of such an effect and we do not know whether there is a quantitative upper limit of the enhancement factor. Our works were devoted to elucidating such issues.

While the slow light effect is obviously due to the dispersion of a photonic crystal along the direction of light propagation, namely the longitudinal direction; the superprism effect, apparently related to angular dispersion, will be shown to have an elusive dependence on the longitudinal dispersion as well. Some subtle connections and distinctions between the slow-light effect and the superprism effect will be revealed through our physical analysis. This allows us to examine these two effects under a common theoretical framework based on photonic crystal dispersion function, $\omega(\mathbf{k})$. An example will be used to illustrate the value of this synergistic theoretical perspective on the slow-light effect and superprism effect, two seemingly distinctive phenomena in photonic crystals. Dispersive effects are frequently accompanied by high loss and/or narrow bandwidth. We will discuss these issues for both longitudinal and angular dispersions. Similar to the bandwidth-delay product for the longitudinal dispersion, we will introduce a simple, yet fundamental, limit that governs the bandwidth and sensitivities of the angular dispersion.

2. THE SLOW LIGHT EFFECT AND THE LONGITUDINAL DISPERSION

2.1. The Origin of the Slow Light Effect in Photonic Crystals

The group velocity of light can be slowed down in various types of photonic crystal structures, especially when the wavelength of light approaches a bandedge. Two common cases shall be considered.

(1) For a “bulk” photonic crystal, such a bandedge typically appears around some high symmetry points in reciprocal space.⁸ As such, in real space, slow light propagation

typically occurs along certain high symmetry axes of a photonic crystal.

(2) For a photonic crystal waveguide (PCW) composed of a line-defect, generally the waveguide is already aligned with a high symmetry axis of the photonic crystal lattice (for example, the ΓK axis of a hexagonal lattice⁹). The original lattice periodicity remains along the longitudinal direction of the waveguide. This results in a one-dimensional (1D) photonic band structure with a maximum or minimum (or other types of extrema) at the 1D Brillouin zone (BZ) boundary $\beta = \pi/a$, where β is the propagation constant of the photonic crystal waveguide in question. Since the dispersion relation $\omega(\beta)$ is generally a smooth function, an extremum ensures $v_g = d\omega/dk = 0$ at the BZ boundary. Therefore, the periodicity along the longitudinal direction dictates that a vanishing group velocity must exist in such a photonic crystal waveguide.

2.2. Applications of the Slow Light Effect

The slow group velocity of light renders the phase shift in a photonic crystal structure more sensitive to refractive index changes.¹ Generally, as the refractive index changes, the dispersion relation of a photonic crystal or a PCW will be shifted by a certain amount $\Delta\omega = \sigma\omega(\Delta n/n)$ along the frequency axis. Here ω is the frequency of light, n is the refractive index, and σ is a factor typically on the order of unity. In many cases, we are interested in a small frequency range where σ can be regarded as a constant. In the case of a PCW, the factor σ can be interpreted as the fraction of the mode-energy in the waveguide core region. For a given wavelength, the propagation constant changes as $\Delta\beta = \Delta\omega/v_g$. Therefore, the phase shift induced by a refractive index change of Δn is given by¹

$$\Delta\phi = \Delta\beta L = \frac{n_g \sigma \Delta n}{n} \frac{2\pi L}{\lambda} \quad (1)$$

where $n_g = c/v_g$ is the group index. Evidently, a slow group velocity (or a high n_g) enhances the phase shift significantly.

To exploit such a significant slow-light enhancement, a number of physical mechanisms^{10–14} have been employed to change the refractive index and actively tune the phase shift in a photonic crystal waveguide. Here we briefly review our works on thermo-optic and electro-optic tuning of the phase shift for optical modulation and switching applications. Many common semiconductor materials, such as silicon and GaAs, have appreciable thermo-optic coefficients ($dn/dT > 10^{-4}/K$). As such, they are suitable for making thermo-optically tunable slow-light photonic crystal devices. In these devices, thermal expansion also contributes to the tuning of the phase shift. In many cases, these two effects add upon each other to produce a larger phase shift. We have demonstrated thermo-optic tuning in a photonic crystal waveguide Mach-Zehnder interferometer with an interaction

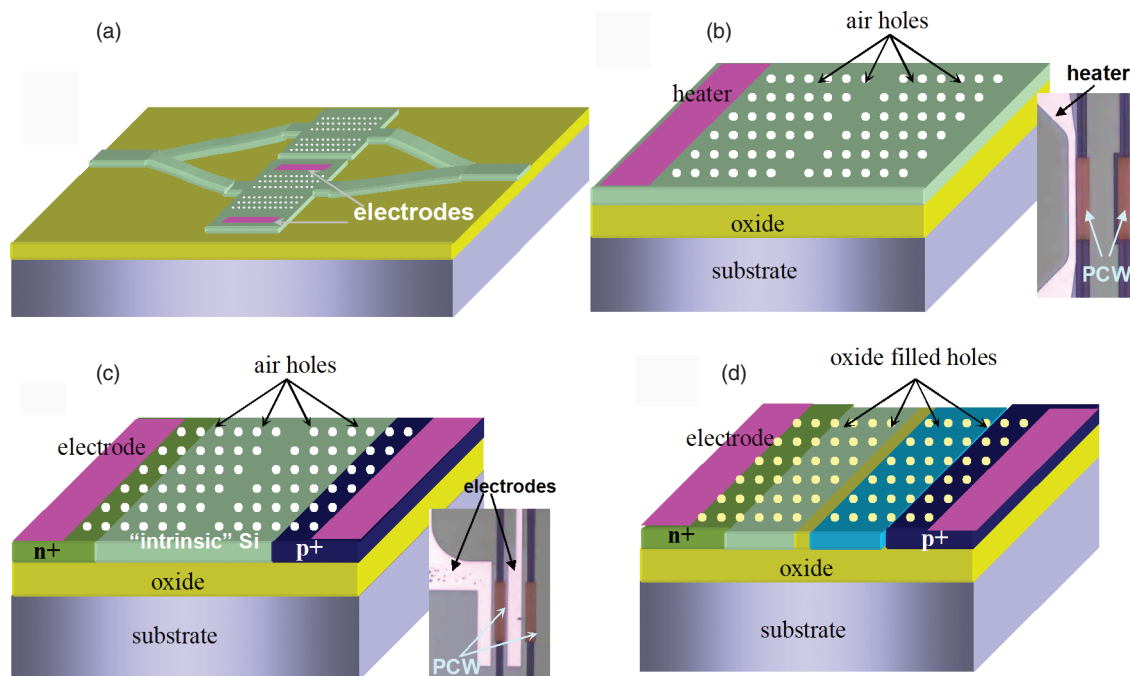


Fig. 1. Photonic crystal waveguide based Mach-Zehnder modulators/switches. (a) A generic photonic crystal waveguide based Mach-Zehnder interferometer with one active tuning arm on a silicon-on-insulator wafer. (b) cross-sectional schematic of an active arm with thermo-optic tuning (inset: micrograph of a thermo-optic device—top view); (c) cross-sectional schematic of an active arm with an embedded silicon p-i-n diode for electro-optic modulation (inset: micrograph of an electro-optic device—top view); (d) cross-sectional schematic of an active arm with an embedded silicon MOS capacitor for electro-optic modulation.

length of $80\ \mu\text{m}$.¹⁰ A photonic crystal waveguide Mach-Zehnder interferometer with one active arm is schematically illustrated in Figure 1(a). A close-up view of the arm with thermo-optic tuning is shown in Figure 1(b). The device was patterned on a silicon-on-insulator (SOI) wafer using a combination of e-beam nanolithography and photolithography. The switching rise time and fall time were measured to be $19.6\ \mu\text{s}$ and $11.4\ \mu\text{s}$, respectively.

Alternatively, we can electro-optically change the refractive index by carrier injection into silicon. Soref and Bennett found the following relation between the refractive index of silicon and the carrier concentrations for wavelengths near $1.55\ \mu\text{m}$ ¹⁵

$$\Delta n = -[8.8 \times 10^{-22} \Delta N_e + 8.5 \times 10^{-18} (\Delta N_h)^{0.8}] \quad (2)$$

where ΔN_e and ΔN_h are electron and hole concentration changes, respectively. A refractive index change up to $\Delta n \sim 10^{-3}$ can be obtained with $\Delta N_e = \Delta N_h \sim 3 \times 10^{17}\ \text{cm}^{-3}$. We demonstrated the first high-speed photonic crystal waveguide modulator on silicon in 2007.¹¹ A schematic of the active arm of the device is shown in Figure 1(c). The device was made on a silicon-on-insulator wafer through a series of micro- and nano-fabrication processes, including e-beam lithography, photolithography, dry and wet etching, ion implantation, and metal lift-off. The device had a measured modulation bandwidth in excess of 1 GHz, with the lowest driving voltage for high-speed silicon modulators at the time of publication. It

should be noted that the introduction of air holes does not significantly increase the electrical resistance of silicon. In Figure 2, we plot the electrical resistance of a photonic crystal waveguide made in a silicon slab for varying air hole sizes and varying number of rows of air holes. In the simulation, the electrical contact pads were assumed to be placed at a fixed separation about $10\ \mu\text{m}$ along the sides of the photonic crystal waveguide. The contact resistance

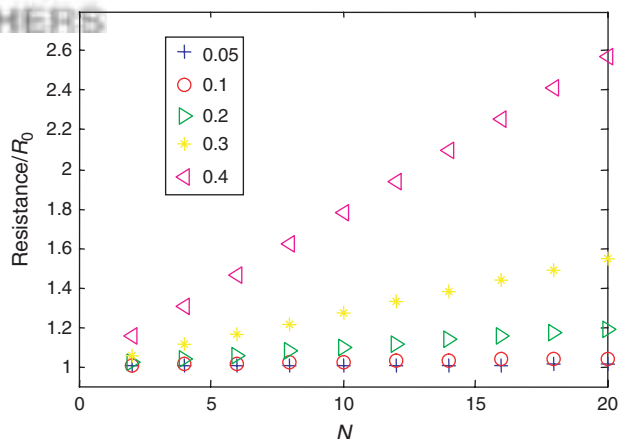


Fig. 2. Electrical resistances for a photonic crystal waveguide in a silicon slab. The horizontal axis indicates the total number of rows of air holes between two electrodes separated by $10\ \mu\text{m}$. The different curves correspond to different values of r/a ($0.05 \sim 0.4$), where r is the radius of the air holes and a is the lattice constant ($\sim 400\ \text{nm}$). The resistance values are normalized by the original slab resistance, R_0 .

can generally be neglected. Evidently, the electrical resistance may increase about 2.6 times for a large hole radius and for a large number of holes, but the value generally remains on the same order of magnitude as the original silicon slab. In our experiments, we also did not observe an order-of-magnitude change of the resistance after the photonic crystal structures were etched in a silicon slab. The resistance values in Figure 2 were computed by 2D finite element method for the DC case.

In our 2007 work, we also derived the AC injection current density for a Si modulator based on a forward-biased p-i-n diode^{11,12}

$$j = 2qw_i\Delta Nf \quad (3)$$

where w_i is the intrinsic region width of the p-i-n diode, and f is the modulation frequency. Combining Eqs. (2) and (3), we obtained a minimum AC current density of 10^4 A/cm² for high speed (>1 GHz) modulation in a typical SOI waveguide. In addition, we showed that due to the non-ideal diode I - V relation $I \sim \exp(qV/2k_B T)$ at high injection, it is possible to limit (or “lock”) the injected carrier concentrations to around $\Delta N \sim 3 \times 10^{17}$ cm⁻³ for a diode with proper doping levels and under normal forward bias conditions. This ensures that the silicon modulator naturally works under the most desired electro-optic state. In a follow-up work, we predicted that an RF power consumption of less than 50 mW is possible for 10 GHz silicon modulators.¹² Subsequently, IBM demonstrated a 10 GHz silicon modulator with 50 mW RF driving power in the forward bias mode;¹⁶ MIT Lincoln Laboratory also reported similar power consumption for 10 GHz silicon modulators.¹⁷ These results affirmed the value of Eq. (3) in designing high speed silicon modulators.

We also developed a metal-oxide-semiconductor (MOS) type photonic crystal waveguide modulator, as illustrated in Figure 1(d). A MOS capacitor can be embedded into a slot photonic crystal waveguide, where the slot is filled with oxide.¹⁸ In such a waveguide, there exist two enhancement effects: the slow light effect, and the field boost inside the low-dielectric slot due to the continuity of surface-normal displacement vector component. Our most recent results show that such a configuration can help reduce the power consumption of a silicon MOS modulator.¹⁹

As mentioned earlier, the slow light effect can also occur in a “bulk” photonic crystal without intentionally introducing line-defects. Such a configuration has been explored in photonic crystal slabs made of conventional electro-optic materials such as LiNbO₃.²⁰

2.3. Loss-Limited Effect

The practical application of the slow-light effect is primarily limited by optical loss. For most practical applications, group velocity values of 100 or less have been currently considered. Further slowing down light causes the optical

loss to increase significantly. To understand the slow light effect, a close examination of the accompanying optical loss is warranted.

The total insertion loss of a photonic crystal waveguide is given by

$$Loss(\text{dB}) = 10 \log_{10} C_1 + 10 \log_{10} C_2 - \alpha L \quad (4)$$

where C_1 and C_2 are the coupling efficiencies at the input and output end of the photonic crystal waveguide, and α is the propagation loss coefficient (in the unit of dB/cm). Note that in our definition, $0 < C_1 < 1$, $0 < C_2 < 1$, $\alpha > 0$. The loss coefficient can be expressed as

$$\alpha = \alpha_1 n_g + \alpha_2 n_g^2 + \dots \quad (5)$$

where the first term can be attributed to absorption and out-of-plane scattering by random imperfections in the photonic crystal waveguide, and the second term can be attributed primarily to back-scattering (due to random imperfections) into the reverse propagating mode with an identical group index.

Several works^{21,22} have discussed the scaling of scattering loss theoretically. Here we give a proof of Eq. (5) that does not invoke the detailed solution of the waveguide equation. We consider the scattering process due to random imperfections in a photonic crystal waveguide. For any scattering event of interest, the initial state must be a guided mode, which we assume has a propagation constant β . The final state can be a guided mode or a radiation mode. For a line-defect waveguide formed in a photonic crystal slab, the radiation modes propagate out of the plane. Assume the scattering amplitude between an arbitrary initial state β and a final state k is $T_{k\beta}$. In any physical situation, the incoming light is always a wave-packet with a continuous distribution of β values, although often the β values are within a narrow range centered around β_0 . The scattering coefficient for such a wave-packet is roughly

$$\begin{aligned} S_{\text{eff}} &\sim \int d\beta \sum_k |T_{k\beta}|^2 \delta(\omega_f - \omega_i) \\ &\sim \int d\beta \int dk'^3 |T_{k'\beta}|^2 \delta(\omega_f - \omega_i) \\ &\quad + C_B \int d\beta \int d\beta' |T_{\beta'\beta}|^2 \delta(\omega_f - \omega_i) \end{aligned} \quad (6)$$

where k' represents a final radiation mode, β' represents a final guided mode, and C_B is a constant. The factor $\delta(\omega_f - \omega_i)$ ensures that the frequencies of the initial and final states are the same (energy conservation). The $(1/v_g)$ factor will arise naturally from each integration of an arbitrary function with respect to β or β'

$$\int f(\beta) d\beta = \int f[\beta(\omega)] d\omega \left| \frac{d\beta}{d\omega} \right| = \frac{1}{v_g} \int f[\beta(\omega)] d\omega. \quad (7)$$

Therefore, we find

$$\begin{aligned} S_{\text{eff}} &\sim \tilde{T}_1/v_g(\beta) + \tilde{T}_2/[v_g(\beta)v_g(\beta')] \\ &= \tilde{T}_1/v_g(\beta) + \tilde{T}_2/v_g^2(\beta). \end{aligned} \quad (8)$$

where $\tilde{T}_1$ and $\tilde{T}_2$ are some constants. The second line in Eq. (8) follows from $\beta' = -\beta$, according to energy conservation in typical photonic crystal waveguides. Note that a similar factor $1/v_g(k')$ may arise from the integration $\int dk^3$ as well. However, the group velocity, $v_g(k')$, of a radiation mode never vanishes. Therefore, this factor has no significance here and is absorbed into $\tilde{T}_1$. Thus, Eq. (5) is proved.

The above derivation clearly shows that the out-of-plane scattering has only one n_g factor because only the initial state is a slow-light state, whereas the backscattering process has a n_g^2 factor because both the initial and final states are slow-light states. We would be tempted to assume that the value of the second integral in Eq. (6) is much smaller than that of the first, because there are a large number of radiation modes that satisfy the energy conservation whereas only one backward guided mode does so. In other words, the total “scattering cross-section” of all radiation modes could be much larger than that of the backward guided mode. However, a general, rigorous proof is needed.

Experimentally, quantitative evaluation of these scaling laws has been elusively difficult and the reported loss dependences^{23–26} vary between $v_g^{-1/2}$ and v_g^{-2} , as discussed in Ref. [27]. It should be clarified that because the scattering events occur statistically uniformly over a given distance, the scattering loss should have the general form $\alpha = -(1/L) \log T_{\text{prop}} \sim v_g^{-\tau}$, not $T_{\text{prop}} \sim v_g^{-\tau}$. On the other hand, the coupling loss coefficients should have the form $C_1, C_2 \sim v_g^\delta$, where $\delta = 1$ for a normally (abruptly) terminated photonic crystal waveguide (see Eq. (15) and related discussions in Section 3.1). Therefore, for a normally terminated photonic crystal waveguide, the total insertion loss is given by

$$\text{Loss(dB)} \approx B_0 - 10 \log_{10} n_g - (\alpha_1 n_g + \alpha_2 n_g^2) L \quad (9)$$

where B_0 is a constant that gives the “baseline” insertion loss.

Here we list several issues in characterizing the optical loss in the slow light regime against Eqs. (4) and (5), or (9): (1) the unknown relative magnitudes of α_1 and α_2 ; (2) the difficulties of separating the coupling loss and propagation loss in experiments; (3) the proper application of the scaling laws of the propagation and coupling losses. It is our feeling that systematic and careful experimental studies over a wide range of waveguide parameters must be performed before a conclusive statement can be put forth regarding the optical loss in the slow-light regime.

3. THE SUPERPRISM EFFECT AND THE ANGULAR DISPERSION

When a light beam is incident upon a photonic crystal surface, the refraction angle inside the photonic crystal could be 500 times more sensitive to the wavelength perturbation

than in a conventional medium.² This so called superprism effect is a manifestation of the strong angular dispersion of a photonic crystal.

Significant progress has been made in investigating the superprism effect in the first ten years after its initial discovery. A number of experiments have demonstrated the potential of the superprism effect in wavelength division multiplexing, beam steering, and sensing applications.^{3–5, 7, 28, 29} Nonetheless, many fundamental questions remained unanswered: (1) How to express the angular dispersion or angular sensitivity of a photonic crystal in terms of basic parameters of a photonic band structure as we have seen in the slow-light effect? (2) Is there an ultimate limit of the angular sensitivity of a photonic crystal? (3) If there is a limit, what are the limiting factors?

To build a foundation for the solution of these problems, we developed a rigorous theoretical framework to compute the transmission and reflection coefficients for refraction across a photonic crystal surface in a 2005 work.³⁰ A parallel work was reported by a group at the University of Toronto in the same period.^{31, 32} Subsequently, we developed the first theory to systematically address the aforementioned general questions in a 2008 work.³³ While the key parameter for tuning the longitudinal dispersion is the group velocity, a new parameter, the curvature of the dispersion surface, must be introduced to describe the angular dispersion. Here the dispersion surface refers to the constant-frequency surface in reciprocal space. This curvature can be calculated directly from the dispersion relation $\omega(\mathbf{k})$, which also represents the photonic band structure. With this theory, we can now directly express the sensitivity of the superprism effect in terms of $\omega(\mathbf{k})$ and explore the fundamental limiting factors of the superprism effect.

In this section, we will briefly introduce our theoretical framework for the superprism effect. Then we will separately discuss two types of superprism effects: the slow-light induced angular dispersion effect and the “pure” angular dispersion effect. We will see some critical scalings and limiting factors for these two types of effects.

3.1. Dispersion Surface Curvature and Angular Sensitivity

For convenience, we will consider a 2D photonic crystal and the TM mode of light (magnetic field in the plane). However, our discussion is applicable to other cases. First, we introduce the concept of the dispersion surface curvature. The dispersion surface at an arbitrary circular frequency, $\omega_0 = 2\pi c/\lambda_0$, can be described by

$$\omega(k_x, k_y) = \omega_0$$

This equation, which gives k_y as an implicit function of k_x , can be reformulated into an explicit form

$$k_y = \kappa(k_x) \quad (10)$$

where we have omitted the parameter ω_0 . Generally, the curvature of a curve on the $k_x - k_y$ plane is given by³³

$$\zeta \equiv \frac{d^2 k_y}{dk_x^2} \left/ \left[1 + \left(\frac{dk_y}{dk_x} \right)^2 \right]^{3/2} \right. \quad (11)$$

where the derivatives can be calculated from the function given in Eq. (10).

To derive the relation between the curvature of the dispersion surface and the angular dispersion of the photonic crystal, we consider the conservation of tangential wavevector component across the photonic crystal surface for a configuration depicted in Figure 3

$$\frac{1}{c} n_l \omega \sin \alpha = k_{x0} + u \sin \theta - v \cos \theta \quad (12)$$

where α is the incident angle, n_l is the refractive index of the incident medium, θ is the direction of the group velocity (i.e., the beam direction) with respect to the surface normal, u and v are local Cartesian coordinates in the neighborhood of a point, $\mathbf{k}_0 = (k_{x0}, k_{y0})$, on the dispersion surface. Here the local u -axis is parallel to the group velocity, and v -axis is tangential to the dispersion surface.

It can be shown from Eq. (12) that the sensitivity of the beam angle to wavelength change (i.e., the angular dispersion) is given by³³

$$\left| \frac{d\theta}{d\lambda} \right| = \left| \frac{2\pi\zeta}{\lambda^2 \cos \theta} (n_g \sin \theta - n_l \sin \alpha) \right| \quad (13)$$

In addition, the sensitivity to refractive index perturbation is given by

$$\left| \frac{d\theta}{dn_a} \right| = \left| -\frac{\zeta \sin \theta}{c \cos \theta} \left(\frac{\partial \omega}{\partial n_a} \right)_k n_g \right| \quad (14)$$

In Eqs. (13) and (14), the quantities, ζ , $1/\cos \theta$ (note $\tan \theta = \sin \theta / \cos \theta$), and n_g are the only three factors that can grow several orders of magnitude compared to a conventional medium, which can result in significant enhancements of angular dispersion/sensitivities as observed in prior superprism experiments.

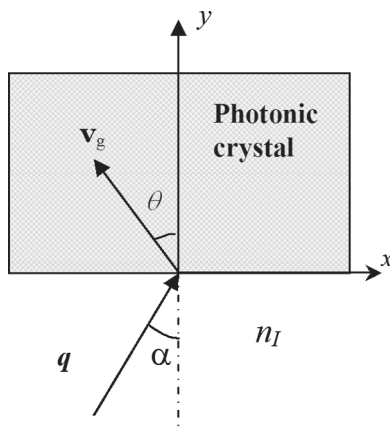


Fig. 3. Schematic of a simple configuration for the superprism effect.

However, it turns out that the optical transmission across the photonic crystal surface is given by³⁰

$$T \propto |t|^2 \varepsilon_{\text{em}} v_g \cos \theta \quad (15)$$

where ε_{em} is the cell-averaged mode energy density and t is the complex coupling amplitude of the mode in question. Evidently, while larger values of n_g or $1/\cos \theta$ will help enhance the angular dispersion and angular sensitivities, they will also inevitably suppress the optical transmission. Therefore, this type of enhancement will eventually be limited by the maximal optical loss that can be tolerated in a particular application. Note that light propagation inside the photonic crystal may further induce significant optical loss in the slow-light regime, in addition to the surface transmission loss given in Eq. (15). The total loss may be handled by a theory similar to the discussion following Eq. (5). On the other hand, enhancing the angular sensitivity through large ζ values will not entail high optical loss, therefore is highly preferred in a wide range of applications.

3.2. Slow-Light Induced Strong Angular Dispersion

Although Eqs. (13) and (14) obviously indicate a linear dependence of the angular dispersion and angular sensitivity on the group index, a casual numeric analysis without the knowledge of Eqs. (13) and (14) would yield a deceptively stronger enhancement in the slow-light regime. Consider the following approximate mode dispersion near a photonic bandedge

$$\omega = \omega_0 - b(k_x^2 + k_y^2) \quad (16)$$

where ω_0 is the bandedge frequency and b is a constant.

The two key parameters for the slow-light effect and the superprism effect have the following frequency dependence near the bandedge

$$\begin{aligned} n_g &= c/[2\sqrt{b(\omega - \omega_0)}] \\ \zeta &= \sqrt{b/(\omega - \omega_0)} \end{aligned} \quad (17)$$

Evidently, the group index and the curvature diverge at the same rate as ω approaches the bandedge ω_0 . It is straightforward to show

$$\begin{aligned} \frac{d\theta}{d\lambda} &\sim \frac{1}{\omega - \omega_0} \\ \frac{d\theta}{dn_a} &\sim \frac{1}{\omega - \omega_0} \end{aligned} \quad (18)$$

A straightforward numeric calculation should find that near the bandedge, when the group index increases 10 times, the angular dispersion and angular sensitivities would increase 100 times. Thus, if we did not have the knowledge of the analytic form of Eqs. (13) and (14), we would be tempted to conclude $d\theta/d\lambda \propto n_g^2$ and

$d\theta/dn_a \propto n_g^2$ in this particular case. However, the above analysis shows that the proper dependence should be $d\theta/d\lambda \propto n_g \zeta$ and $d\theta/dn_a \propto n_g \zeta$ because n_g and ζ diverge at the same rate. Although the above analysis is based on a specific photonic band structure described by Eq. (16), a general asymptotic analysis, given in Section IIA of Ref. [33], indicates that such a slow-light induced strong angular dispersion due to equal diverging rate of n_g and ζ exists in a wider range of slow-light scenarios.

3.3. “Pure” Angular Dispersion Effect: Bandwidth Limited

We have found that the dispersion surface can exhibit ultra-high curvature values in the vicinity of certain high-symmetry points in the Brillouin zone (BZ) without the presence of the slow-light effect.³³ An example is the K point of a triangular lattice. Some photonic bands have a double degeneracy at this point. Approaching such a doubly degenerate point, the curvature of the dispersion surface tends to infinity whereas the group velocity approaches a non-zero constant. Therefore, we can enjoy the benefit of the high dispersion and high sensitivity, according to Eqs. (13) and (14), without worrying about the high optical loss that would occur in the slow-light case.

In this case, the scalings of the group index and curvature differ from the slow-light induced superprism effect,

$$\begin{aligned} n_g &\rightarrow \text{const} \\ \zeta &\rightarrow 1/(\omega - \omega_0) \end{aligned} \quad (19)$$

where ω_0 is the frequency of the doubly degenerate point, where the curvature is singular. Interestingly, according to Eqs. (13) and (14), the overall scaling of the angular dispersion and angular sensitivities with frequency perturbation, $\Delta\omega = \omega - \omega_0$, in this case remain the same as in Eq. (18). However, the optical loss is almost constant in this case, independent of the angular dispersion values.

With the optical loss no longer being a limiting factor, the angular dispersion and angular sensitivities can now be enhanced to much higher values until it encounters some other limits. Now we present a fundamental angular sensitivity-bandwidth limit similar to the bandwidth-delay limit for the longitudinal dispersion. Assume the angular dispersion is maintained at a value above $(d\theta/d\lambda)_0$ over a spectral range of BW (unit: nm). Because the maximum beam steering range can not exceed 180° (no backward propagation is physically possible), we find

$$(d\theta/d\lambda)_0 \times BW < 180^\circ \quad (20)$$

This relation is the *angular dispersion correspondent of the bandwidth-delay product*.

Therefore, the maximum bandwidth for a sustained high sensitivity $(d\theta/d\lambda)_0$ is

$$BW < 180^\circ / (d\theta/d\lambda)_0 \quad (21)$$

For example, a sensitivity of $100^\circ/\text{nm}$ can be sustained over a bandwidth less than 1.8 nm, and a sensitivity of $1000^\circ/\text{nm}$ can be sustained over a bandwidth less than 0.18 nm. For a laser having 1pm linewidth (~ 100 MHz), these two cases may allow for 1800 and 180 wavelength tuning points, respectively, which are reasonable for practical applications. These performance parameters are possible with the existing laser technologies. Lastly, we should keep in mind that not all applications require a wide bandwidth. There are some applications that can benefit from a high angular dispersion/sensitivity in a narrow bandwidth.

A detailed theoretical analysis based on group theory shows that such types of “pure” angular dispersion originates fundamentally from symmetry induced degeneracy in photonic band structures. Furthermore, such types of symmetry-induced enhancement of angular sensitivities can only occur in 2D and 3D photonic crystals, but not in 1D photonic crystals. Discovering such a crystal-symmetry induced effect exemplifies the effectiveness of utilizing the solid-state physics paradigm to shed new light on the study of periodic dielectric structures, which is the central theme of photonic crystal research.³⁴ In passing, we note that the rigorous theory for computing the transmission of each photonic crystal mode³⁰ has been extended to gratings,³⁵ which can be regarded as monolayer photonic crystals, and 3D photonic crystals.³⁶ Formulas similar to Eq. (15) can be used to assess the optical loss in the 1D and 3D cases as well.

Before we conclude this section, we would like to mention that there are a number of applications for the superprism effect. Different applications may have additional limiting factors specific to themselves. For example, for the widely studied wavelength demultiplexer application, the beam width divergence is an additional limiting factor specific to this application. Fortunately, recent works^{7,37} have demonstrated a promising method of overcoming this limit. Note that this factor is important only for those applications that require narrow beam width (or spot size) at the receiving end of the superprism. If a sufficiently large beam width (hence a small lateral spread of the wavevector) is used, then this factor is less important.

4. LONGITUDINAL DISPERSION VERSUS ANGULAR DISPERSION: A DIRECT COMPARISON

The analyses in two preceding sections show that the two key parameters, v_g and ζ , of the slow-light effect and the superprism effect are entirely determined by the dispersion function $\omega(k_x, k_y)$. In other words, we may say that these two effects are manifestations of the longitudinal and angular characteristics of the dispersion function. In this section, we will unveil some further connections between these two effects through an application example. Here we choose the beam steering application, which intends

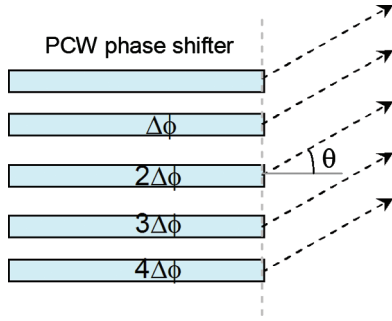


Fig. 4. Schematic of a one-dimensional optical phase array composed of photonic crystal waveguide phase shifters.

to manipulate the direction of a laser beam by changing the refractive indices of materials in certain device structures. Two approaches are considered: (1) an optical phase array³⁸ (OPA) composed of a 1D array of slow-light photonic crystal waveguides as shown in Figure 4; (2) a superprism composed of a 2D photonic crystal. Note that to steer the output beam in free space, the superprism device can not have a flat output surface parallel to the input surface. In this example, we assume the output surface has a semi-circular configuration for simplicity.³

First, we re-write Eqs. (1) and (14) in the following forms:

$$\Delta\phi = L \left(\frac{n_g \sigma}{n} \frac{2\pi}{\lambda} \right) \Delta n \quad (22a)$$

$$|\Delta\theta| = \left| \zeta \tan \theta \left(\frac{n_g \sigma}{n_a} \frac{2\pi}{\lambda} \right) \Delta n_a \right| \quad (22b)$$

The second equation follows from

$$\left(\frac{\partial \omega}{\partial n_a} \right)_k = \sigma_a \frac{\omega}{n_a} \quad (23)$$

where σ_a measures the fraction of mode energy located in the medium a . Note this relation, Eq. (23), is essentially the same as that used in an early derivation of the slow-light enhancement of the phase sensitivity in a photonic crystal waveguide.¹ Therefore, it is not surprising to see that Eqs. (22a) and (22b) share similar factors $(\sigma/n)(2\pi/\lambda)$, which come from $\partial\omega/\partial n$. In the case of a photonic crystal waveguide, n refers to the refractive index of the waveguide core, and σ denotes the fraction of the mode-energy in the core region.¹

More interestingly, a direct comparison of the steering angle sensitivity between the two approaches can be obtained from Eqs. (22a) and (22b). For an optical phase array, the far-field beam angle θ relates to the phase difference, $\Delta\phi$, between adjacent array elements as follows

$$\sin \theta = \frac{\Delta\phi \lambda}{2\pi d} \quad (24)$$

Therefore, the beam steering sensitivity for a slow-light based optical phase array is given as

$$\Delta\theta = \frac{1}{\cos \theta} \frac{n_g \sigma}{n} \frac{L}{d} \Delta n \quad (25)$$

To simplify the comparison with the superprism effect, we assume $\sigma = \sigma_a$, and $n = n_a$. Then the ratio of the beam angle changes in the two cases is given by

$$\left| \frac{\Delta\theta_{SP}}{\Delta\theta_{SL}} \right| = \left| \sin \theta \left(\frac{2\pi d}{\lambda} \right) \frac{\zeta n_{g,SP}}{L n_{g,SL}} \right| \quad (26)$$

where SP denotes the superprism effect, and SL denotes the slow-light effect. In most optical phase arrays, the waveguide spacing, d , is on the order of the wavelength, λ . If we assume $\sin \theta > 0.1$ and note the 2π factor in Eq. (26), these two factors have an overall contribution on the order of unity. Therefore, the difference between $\Delta\theta_{SP}$ and $\Delta\theta_{SL}$ primarily comes from the terms, (ζ/L) and $(n_{g,SP}/n_{g,SL})$. Note the curvature ζ also has the dimension of length.

As a numeric example, we consider a silicon photonic crystal waveguide with $n_{g,SL} \sim 30$, and $\Delta n \sim 10^{-3}$. This generally requires a waveguide length on the order of $L = 100 \mu\text{m}$ to achieve a phase shift of 2π at $\lambda = 1.55 \mu\text{m}$. Note that to extend the waveguide length far beyond this value to achieve larger $\Delta\phi$ will cause multiple side-lobes and is not desired for many practical OPA beam steering applications. On the other hand, it is relatively easy to get $\zeta \gg 100 \mu\text{m}$ in a properly designed photonic crystal superprism. For example, we find $2\pi\zeta/\lambda > 10^3$ for a hexagonal photonic crystal with $n_g \sim 7$.³³ In this particular example, the ratio in Eq. (26) is around $3 \sin \theta$. For moderate θ values, the beam steering efficiencies due to the two effects are roughly on the same order of magnitude. A more detailed investigation is beyond the scope of this work.

5. CONCLUSION

In this paper, we have discussed the slow-light effect and the superprism effect in one synergistic perspective based on dispersions. We give rigorous analysis of the phase shift sensitivities, angular sensitivities, optical loss, and bandwidth for these effects in fairly general situations. Particularly, a rigorous proof of the scaling of the scattering loss in the slow light regime is given. The general relations and trends that we have obtained regarding these important parameters provide an important guide for further experiments to verify these effects and to explore new applications.

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